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# Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects

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**Abstract**

The heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a magnetic field is numerically studied, by taking into account the diffusion-thermo (Dufour) and thermal-diffusion (Soret) effects. The governing partial differential equations are transformed into a set of coupled differential equations, which are solved numerically using a finite difference method. Dimensionless velocity, temperature and concentration profiles are presented graphically for various values of the magnetic number $M$ and Lewis number $Le$, and for fixed values of the Dufour number $D_f$, Soret number $S_r$ and buoyancy number $N$. Three cases are considered and presented in tables, for the local Nusselt number and local Sherwood number corresponding to $Le = 1$ and various values of $M$, $N$, $D_f$ and $S_r$. Increasing the value of $M$ increases the local Nusselt number and local Sherwood number.


## 1. Introduction

Coupled heat and mass transfer by natural convection in a fluid-saturated porous medium has attracted considerable attention in the last years, due to many important engineering and geophysical applications. Recent books by Nield and Bejan [1] and Ingham and Pop [2,3] present a comprehensive account of the available information in the field.

Thermal diffusion, also called thermodiffusion or Soret effect [4] corresponds to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient. On the other hand, the Soret effect has been also utilised for isotope separation and in mixture between gases with very light molecular weight, such as $H_2$ or He, and of medium molecular weight, such as $H_2$ or air.

In many studies Dufour and Soret effect are neglected, on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. There are however, exceptions. Eckert and Drake [5] present several cases when the Dufour effect cannot be neglected. Benano-Melly et al. [6] have analyzed the problem of thermal diffusion in binary fluid mixtures, lying within a porous medium and subjected to a horizontal thermal gradient. The onset of Soret-driven convection in an infinite cell filled with a porous medium saturated by a binary fluid was studied by Sovran et al. [7]. Thermal-diffusion (Soret) and difusion-thermo (Dufour) effects have been found to appreciably influence the flow field in free convection boundary-layer over a vertical surface embedded in a porous medium [8].

There has been a considerable interest in studying the effect of a magnetic field on natural convection heat and mass transfer in porous media. In a recent paper Cheng [9] used an integral method to study the natural convection heat and mass transfer from vertical plates embedded in in electrically conducting fluid saturated porous media with constant surface temperature concentration. The application of a transverse magnetic field normal to the flow direction was shown to decrease the Nusselt number and Sherwood number.

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**Nomenclature**

| | |
|---|---|
| $C$ | concentration |
| $C_p$ | specific heat at constant pressure |
| $C_s$ | concentration susceptibility |
| $D_f$ | Dufour number |
| $D_m$ | mass diffusivity |
| $f$ | dimensionless stream function |
| $H_0$ | magnetic field intensity |
| $K$ | Darcy permeability |
| $k_T$ | thermal diffusion ratio |
| $Le$ | Lewis number, $\alpha_m/D_m$ |
| $M$ | magnetic parameter |
| $N$ | sustentation parameter |
| $Ra_x$ | local Rayleigh number |
| $u$, $v$ | Darcian velocities in the $x$ and $y$-direction, respectively |
| $Sh$ | Sherwood number |
| $T$ | temperature |
| $x$, $y$ | Cartesian coordinates normal to the plate and along it, respectively |
| $\alpha_m$ | thermal diffusivity |
| $\beta_T$ | coefficient of thermal expansion |
| $\beta_C$ | coefficient of concentration expansion |
| $\phi$ | dimensionless concentration |
| $\mu$ | viscosity |
| $\mu_e$ | magnetic permeability |
| $\nu$ | kinematic viscosity |
| $\theta$ | dimensionless temperature |
| $\sigma$ | electrical conductivity |
| $\rho$ | density |
| $\psi$ | stream function |
| *Subscripts* | |
| w | condition at wall |
| $\infty$ | condition at infinity |
| *Superscript* | |
| $'$ | differentiation with respect to $\eta$ |

The objective of this paper is to study simultaneous heat and mass transfer by natural convection from a vertical flat plate embedded in electrically conducting fluid saturated porous medium, using Darcy–Boussinesq model, including Soret and Dufour effects. The effects of the governing parameters on the heat and mass transfer are analyzed.

## 2. Analysis

Consider the natural convection in a porous medium saturated with a Newtonian fluid bounded by a vertical flat plate with constant wall temperature $T_w$ and constant wall concentration $C_w$. The temperature and concentration of the ambient medium are $T_\infty$ and $C_\infty$, respectively, where $T_w > T_\infty$ and $C_w > C_\infty$. The $x$-coordinate is measured along the plate from its leading edge, and the $y$-coordinate normal to it. Several assumptions are used throughout the present paper: (a) the fluid and the porous medium are in local thermodynamic equilibrium; (b) the flow is laminar, steady-state and two-dimensional; (c) the porous medium is isotropic and homogeneous; (d) the properties of the fluid and porous medium are constants; (e) the Boussinesq approximation is valid and the boundary-layer approximation is applicable.

In-line with these assumptions, the governing equations describing the conservation of mass, momentum, energy and concentration can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\left(1 + \frac{K\sigma\mu_e^2 H_0^2}{\mu}\right) = \frac{gK}{\nu}[\beta_T(T - T_\infty) + \beta_C(C - C_\infty)] \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \frac{D_m}{C_s}\frac{k_T}{C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where $\alpha_m$ and $D_m$ are the thermal diffusivity and mass difffusivity, $C_p$ and $C_s$ are the specific heat at constant pressure and concentration susceptibility, $k_T$ is the thermal diffusion ratio, $\sigma$, $\mu_e$ and $H_0$ are electrical conductivity, magnetic permeability and magnetic field intensity, respectively. Other quantities are defined in the nomenclature.

The boundary conditions of the problem are

$$y = 0 : v = 0, \quad T = T_w, \quad C = C_w \tag{5a}$$

$$r \to \infty : u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \tag{5b}$$

We use the similarity variables proposed by Cheng and Mynkowycz [10] and used then by Bejan and Khair [11]

$$\psi = \alpha_m Ra_x^{1/2} f(\eta), \quad \theta = (T - T_\infty)/(T_w - T_\infty),$$
$$\phi = (C - C_\infty)/(C_w - C_\infty), \tag{6}$$

$$\xi = \frac{y}{x}Ra_x^{1/2}$$

where the stream function $\psi$ is defined in the usual way

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

and $Ra_x = gK\beta(T_w - T_\infty)x/(\nu\alpha_m)$ is the local Rayleigh number. The governing equations become

$$f' = -\theta - N\phi \tag{8}$$

$$\theta'' - \frac{1}{2}f\theta' + D_f\phi'' = 0 \tag{9}$$

$$\frac{1}{Le}\phi'' - \frac{1}{2}f\phi' + S_r\theta'' = 0 \tag{10}$$

where $M$ is the magnetic parameter, defined as

$$M = \frac{K\sigma\mu_e^2H_0^2}{\mu} \tag{11}$$

$Le$, $D_f$ and $S_r$ are Lewis, Dufour and Soret numbers, respectively

$$Le = \frac{\alpha_m}{D_m}, \quad D_f = \frac{D_m k_T(C_w - C_\infty)}{C_s C_p \alpha_m (T_w - T_\infty)},$$
$$S_r = \frac{D_m k_T(T_w - T_\infty)}{C_s C_p \alpha_m (C_w - C_\infty)} \tag{12}$$

whilst $N$ is the sustentation parameter

$$N = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)} \tag{13}$$

Table 1
Values of Nusselt and Sherwood numbers in case I

| $M$ | $Le$ | $N$ | $D_f$ | $S_r$ | $Nu_x/Ra_x^{1/2}$ | $Sh_x/Ra_x^{1/2}$ |
|---|---|---|---|---|---|---|
| 0 | 1 | 1.0 | 0.05 | 1.2 | 0.67678 | 0.18354 |
| 1 | 1 | 1.0 | 0.05 | 1.2 | 0.47855 | 0.12978 |
| 2 | 1 | 1.0 | 0.05 | 1.2 | 0.30266 | 0.08209 |
| 0 | 1 | 1.0 | 0.075 | 0.8 | 0.65108 | 0.34150 |
| 1 | 1 | 1.0 | 0.075 | 0.8 | 0.46038 | 0.24148 |
| 2 | 1 | 1.0 | 0.075 | 0.8 | 0.29117 | 0.15273 |
| 0 | 1 | 1.0 | 0.03 | 2.0 | 0.71444 | −0.13597 |
| 1 | 1 | 1.0 | 0.03 | 2.0 | 0.50519 | −0.09869 |
| 2 | 1 | 1.0 | 0.03 | 2.0 | 0.31951 | −0.06241 |
| 0 | 1 | 1.0 | 0.037 | 1.6 | 0.69686 | 0.02339 |
| 1 | 1 | 1.0 | 0.037 | 1.6 | 0.49275 | 0.01654 |
| 2 | 1 | 1.0 | 0.037 | 1.6 | 0.31164 | 0.01047 |
| 0 | 1 | 1.0 | 0.6 | 0.1 | 0.42002 | 0.63313 |
| 1 | 1 | 1.0 | 0.6 | 0.1 | 0.29700 | 0.44769 |
| 2 | 1 | 1.0 | 0.6 | 0.1 | 0.18784 | 0.28315 |

Table 2
Values of Nusselt and Sherwood numbers in case II

| $M$ | $Le$ | $N$ | $D_f$ | $S_r$ | $Nu_x/ Ra_x^{1/2}$ | $Sh_x/ Ra_x^{1/2}$ |
|---|---|---|---|---|---|---|
| 0 | 1 | −0.5 | 0.15 | 0.4 | −0.28512 | −0.23211 |
| 1 | 1 | −0.5 | 0.15 | 0.4 | −0.20169 | −0.16425 |
| 2 | 1 | −0.5 | 0.15 | 0.4 | −0.12839 | −0.10526 |

Table 3
Values of Nusselt and Sherwood numbers in case III

| $M$ | $Le$ | $N$ | $D_f$ | $S_r$ | $Nu_x/Ra_x^{1/2}$ | $Sh_x/Ra_x^{1/2}$ |
|---|---|---|---|---|---|---|
| 0 | 1 | 0.2 | 0.15 | 0.4 | 0.46331 | 0.38100 |
| 1 | 1 | 0.2 | 0.15 | 0.4 | 0.32762 | 0.26942 |
| 2 | 1 | 0.2 | 0.15 | 0.4 | 0.20723 | 0.17044 |
| 0 | 1 | 0.5 | 0.075 | 0.8 | 0.55508 | 0.28764 |
| 1 | 1 | 0.5 | 0.075 | 0.8 | 0.39250 | 0.20339 |
| 2 | 1 | 0.5 | 0.075 | 0.8 | 0.24825 | 0.12866 |
| 0 | 1 | 0.8 | 0.03 | 2.0 | 0.67028 | −0.13736 |
| 1 | 1 | 0.8 | 0.03 | 2.0 | 0.47936 | −0.09712 |
| 2 | 1 | 0.8 | 0.03 | 2.0 | 0.29976 | −0.06142 |

which measures the relative importance of mass and thermal diffusion in the buoyancy-driven flow. We notice that it is positive for thermally assisting flows, negative for thermally opposing flows and zero for thermal-driven flows. Primes denote differentiation with respect to $\eta$.

The transformed boundary conditions are

$$f(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \tag{14a}$$

$$f' \to 0, \quad \theta \to 0, \quad \phi \to 0, \quad \text{as } \eta \to \infty \tag{14b}$$

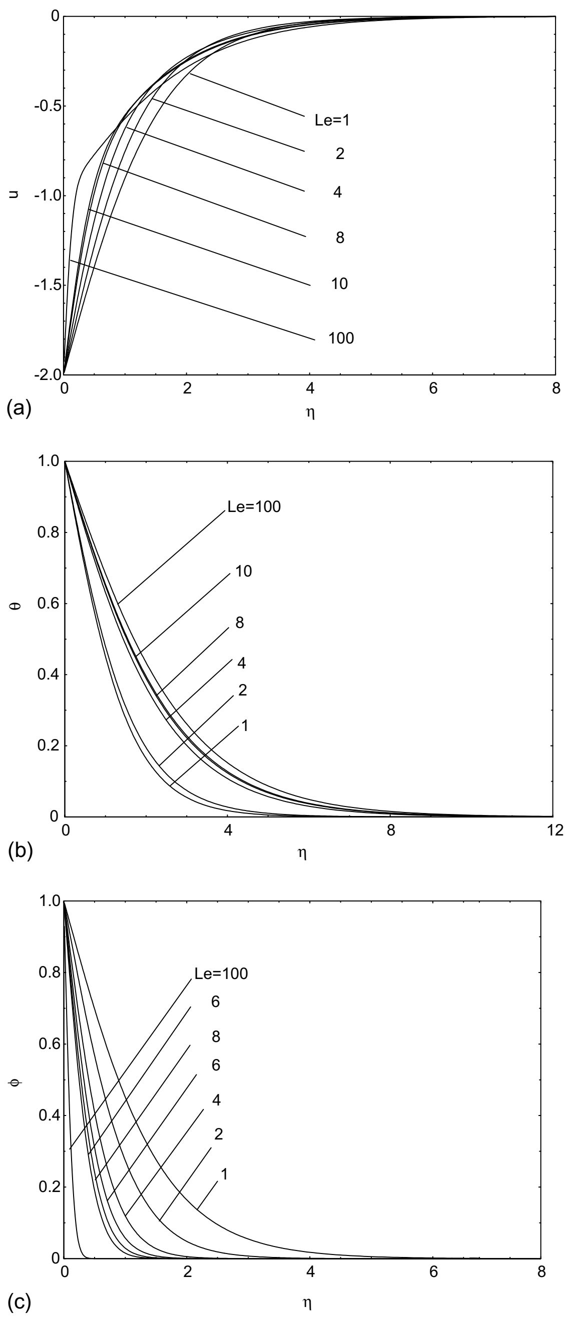


Fig. 1. Variations of (a) velocity, (b) temperature (c) concentration across the boundary layer for $N = 1$, $D_f = 0$, $S_r = 0$ and $M = 0$.

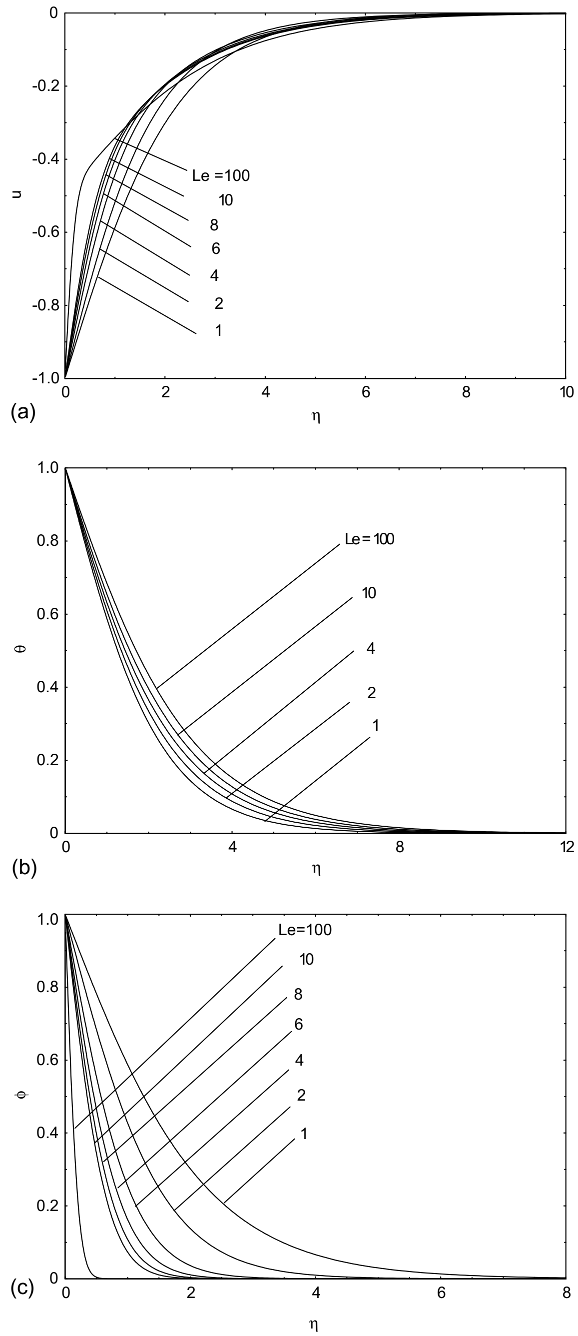


Fig. 2. Variations of (a) velocity, (b) temperature (c) concentration across the boundary layer for $N = 1$, $D_f = 0$, $S_r = 0$ and $M = 1$.

We notice that the problem reduces to that formulated by Bejan and Khair [11] when $M = 0$, $D_f = 0$ and $S_r = 0$. On the other hand, for $M = 0$ our Eqs. (8)–(10) subjected to the boundary conditions (14) reduce to (7)–(10) of Anghel et al. [8].

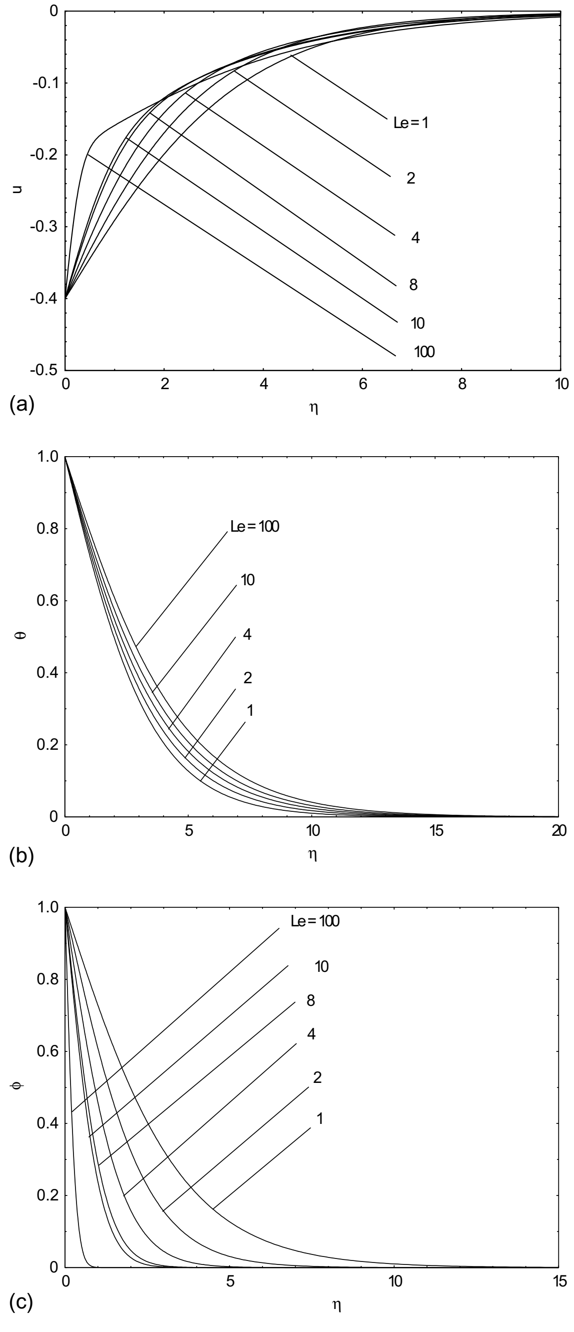


Fig. 3. Variations of (a) velocity, (b) temperature (c) concentration across the boundary layer for $N = 1$, $D_f = 0$, $S_r = 0$ and $M = 2$.

The parameters of engineering interest for the present problem are the local Nusselt number and local Sherwood number, which are given by the expressions

$$Nu_x/Ra_x^{1/2} = -\theta'(0), \quad Sh_x/Ra_x^{1/2} = -\phi'(0) \qquad (15)$$

## 3. Results and discussion

Eqs. (8)–(10) must be solved along with the boundary conditions (14). Since analytical solutions do not exist, one has to use numerical techniques. In this paper a version of the Keller-box method adapted to solve ordinary differential equations was used [12].

The parameters involved in the present problem are $M$, $Le$, $N$, $D_f$ and $S_r$. Three cases are considered here, according to [8]:

- Case I: $Le = 1$, $N = 1$, $(D_f, S_r) = ((0.05, 1.2), (0.075, 0.8), (0.03, 2.0), (0.037, 1.6), (0.6, 0.1))$
- Case II: $Le = 1$, $N = 1$, $D_f = 0.15$, $S_r = 0.4$
- Case III: $Le = 1$, $(N, D_f, S_r) = ((0.2, 0.15, 0.4), (0.5, 0.075, 0.8), (0.8, 0.03, 2.0))$.

In each case, the values of the magnetic parameter $M$ were taken as 0, 1 and 2. Tables 1–3 present local Nusselt and Sherwood numbers calculated for each set of parameters. One can readily remark that, for fixed $Le$, $N$, $D_f$ and $S_r$, $Nu$ and $Sh$ decrease as $M$ increases (if negative values are encountered, the previous assertion holds for absolute values).

Figs. 1–3 show the dimensionless velocity, temperature and concentration for the following values of the parameters: $N = 1$, $D_f = 0$, $S_r = 0$, $M = (0, 1, 2)$ and $Le = (1, 2, 4, 6, 8, 10, 100)$.

We remark that as $M$ increases, the thickness of the hydrodynamic/thermal/concentration boundary layer increases.

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